Circular

RULES – IMPLEMENTATION JANUARY 23, 2020

GENERAL LIABILITY LI-GL-2020-017

MONTANA GENERAL LIABILITY INCREASED LIMIT FACTORS TO BE IMPLEMENTED; EXHIBITS NEWLY PRESENTED IN EXCEL

KEY MESSAGE

The revised increased limit factors represent a combined change of +1.1% from the increased limit factors currently in effect.

BACKGROUND

In circular LI-GL-2019-061, we provided you with information about the General Liability increased limit experience review.

ISO ACTION

We are implementing GL-2020-IPOP1, which revises the increased limits for all Premises/Operations Liability (subline code 334) class tables in Rule 56. of Division Six of the Commercial Lines Manual.

Refer to the attached explanatory material for complete details about the filing.

IMPORTANT NOTE

Change in Format

This circular offers several enhancements for customers. In addition to the PDF version, exhibits are now available in user-friendly Excel format rather than Word. Where possible, exhibits are linked together formulaically to clarify how calculations flow through the analysis and to enable customers to test the effects of different assumptions on the results.

To facilitate this change, the filing has been restructured. All explanatory text, for all sections of the filing, appears first; all exhibits are grouped together and appear thereafter. Exhibits have been labeled as Exhibit 1, Exhibit 2, etc., with the manual rule page exhibit labeled as Exhibit MP. Exhibits MP is provided in a separate Word file while the other exhibits are available in an Excel file. We invite customers to share feedback on this revised format and suggestions for further enhancements by contacting the individuals listed in the Contact Information block.

IMPORTANT NOTE ON RISK LOAD REFLECTION

The increased limit factors in this document incorporate a procedure for reflecting the increased risk or variation in experience associated with higher limit policies in the increased limits ratemaking formula. For all General and Commercial Automobile Liability tables, this procedure generates increased limit factors that are on average (across all state groups) 6.0% higher than the factors would be if calculated without risk load. For this state group, the indicated increased limit factors are on average 5.6% higher for Premise/Operations than such factors would be if calculated without risk load.

The inclusion of risk load in increased limit factors may have implications on basic limit loss cost multipliers. Specifically, assuming industrywide averages and the ISO increased limit factors in this document, the inclusion of risk load may result in additional revenue of 5.6% for Premises/Operations Liability. All sources of revenue, including the revenue resulting from the risk load in these increased limit factors, should be kept in mind when determining loss cost multipliers.

EFFECTIVE DATE

The ISO revision is subject to the following rule of application:

These changes are applicable to all policies written on or after July 1, 2020.

COMPANY ACTION

If you have authorized us to file on your behalf and decide:

- To use our revision and effective date, you are not required to file anything with the Insurance Department.
- To use our revision with a different effective date, to use our revision with modification, or to not use our revision, you must make an appropriate submission with the Insurance Department.

For guidance on submission requirements, consult the ISO State Filing Handbook.

In all correspondence with the Insurance Department on this revision, you should refer to ISO Filing Designation Number GL-2020-IPOP1, NOT this circular number. Communications with the regulator concerning a filing affecting multiple lines of business (i.e., CL, PL, AL filing designation) should specify the line(s) of business that you are addressing.

RATING SOFTWARE IMPACT

No new attributes are being introduced with this revision.

POLICYHOLDER NOTIFICATION

If you decide to implement this revision, you should check all applicable laws for the state(s) to which this revision applies, to determine whether or not a specific policyholder notice requirement may apply. Please note that circular LI-CL-2019-057 contains the ISO Guide To Renewals With Changed Conditions For Commercial Lines, which is available only as a guide to assist participating companies in complying with various conditional renewal statutes or regulations, for the major commercial lines of insurance serviced by ISO. The information in the Guide does not necessarily reflect all requirements or exceptions that may apply, and it is not intended as a substitute for your review of all applicable statutes and regulations concerning policyholder notification.

REVISION DISTRIBUTION

We will issue a Notice to Manualholders with an edition date of 7-20 (or the earliest possible subsequent date), along with any new and/or revised manual pages.

REFERENCE(S)

- LI-CL-2019-057 (12/10/2019) Revised Lead Time Requirements Listing
- LI-GL-2019-061 (03/15/2019) 2019 General Liability Increased Limits Experience Reviewed By Staff

ATTACHMENT(S)

Filing GL-2020-IPOP1

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DATA QUALITY

Statistical plan data reported to ISO is first processed through a system of rigorous automated data verification procedures so that only valid data would be used for ratemaking. Subsequent to this initial data submission review, additional analyses on the statistical plan data involving an even more customized data review for this line were performed by staff. During these processes, various data records were excluded from the review. The ISO staff responsible for this circular also reviewed the data for reasonableness.

ACKNOWLEDGMENT OF ACTUARIAL QUALIFICATIONS

The American Academy of Actuaries' "Qualifications Standards for Actuaries Issuing Statements of Actuarial Opinion in the United States" requires that an actuary issuing a Statement of Actuarial Opinion should include an acknowledgment with the opinion that he/she has met the qualification standards of the AAA. ISO considers this rules filing a Statement of Actuarial Opinion; therefore, we are including the following acknowledgment:

I, David Terné, am a Managing Director of Strategic Actuarial Operations for ISO and I, James Davidson, am an Actuarial Product Director for Commercial Auto and Increased Limits for ISO. We are jointly responsible for the content of this Statement of Actuarial Opinion. We are both members of the American Academy of Actuaries and we meet the Qualification Standards of the American Academy of Actuaries to render the actuarial opinion contained herein.

CONTACT INFORMATION

If you have any questions concerning:

- The actuarial content of this circular, please contact:

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Callers outside the United States, Canada, and the Caribbean may contact us using our global toll-free number (International Access Code + 800 48977489). For information on all ISO products, visit us at www.verisk.com/iso. To keep abreast of the latest Insurance Lines Services updates, view www.verisk.com/ils.

MONTANA
GENERAL LIABILITY INCREASED LIMIT FACTORS

EXECUTIVE SUMMARY

PURPOSE

This document:

- revises increased limit factors (ILFs) for all Premises/Operations Liability classes. These increased limit factors represent a +1.1% change on average from the Premises/Operations increased limit factors currently in effect. We are not revising Products/Completed Operations increased limit factors in this filing.
- provides the analyses used to derive the revised increased limit factors, including a modification in how we incorporate composite-rated risk data into our review.

DEFINITION OF INCREASED LIMIT FACTORS

We publish liability loss costs at the basic limit. The basic limit for General Liability is $100,000/$200,000 (occurrence/aggregate). The loss cost for a given policy limit is the product of the basic limit loss cost and the increased limit factor for that policy limit.

An increased limit factor is the ratio of two sums. The numerator is the cost to the insurer of writing a policy at the desired limit, including the average prospective indemnity, all loss adjustment expense and the risk load. The denominator is the sum of the same quantities at the basic limit. The average prospective indemnity in the published ILFs reflects per occurrence and aggregate limits.

INCREASED LIMITS TABLES

We group classifications with similar increased limits experience into increased limits tables. Premises/Operations has three tables corresponding with low, medium and high loss severity - the tables are 1, 2 and 3, respectively.

INCREASED LIMIT FACTOR CHANGES

The statewide per occurrence increased limit factor changes are:

Premises/Operations		
	Indicated	Selected
Table 1	+0.6%	+0.6%
Table 2	+0.5%	+0.5%
Table 3	+4.1%	+4.1%
TOTAL	+1.1%	+1.1%

In this document, the selected per occurrence factors are the indicated per occurrence factors. We judgmentally adjust some occurrence/aggregate factors developed from the per occurrence factors to maintain consistency between successive policy limits within each table.

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PRIOR ISO REVISION

The most recent Premises/Operations increased limits revision is:

Designation	GL-2019-IALL1
Date Implemented	07/01/2019
Indicated Change	+1.7%
Selected Change	+1.7%
Implemented Change	+1.7%

(The overall General Liability percentage change in filing GL-2019-IALL1 was +1.1%.)

RISK LOAD PROCEDURE

The increased limit factors in this document incorporate a procedure for reflecting the increased risk or variation in experience associated with higher limit policies in the increased limits ratemaking formula. For all General and Commercial Automobile Liability tables, this procedure generates increased limit factors that are on average (across all state groups) 6.0% higher than the factors would be if calculated without risk load. For this state group, the indicated increased limit factors are on average 5.6% higher for Premises/Operations than such factors would be if calculated without risk load.

HISTORICAL SOURCE DATA

For this filing, we used the following data:

- Experience from occurrence-coverage policies for risks subject to Premises/Operations increased limits tables as reported to ISO by companies that filed detailed statistics. This includes excess and umbrella data reported under the Commercial Statistical Plan, which adds greater credibility to the analysis of higher layers. Experience for risks reported in the ISO Annual Call for Excess and Umbrella Policy Claims supplements primary data for pricing higher policy limits.

- Experience for accident years ending December 31, 2004 to December 31, 2017, which were settled during calendar years 2013 to 2017.

Please note that for Premises/Operations, we review the data by state or state group. Only the largest states have sufficient volume to be reviewed individually. We have grouped all other states based on an analysis of their historical distributions. For certain calculations, we use multistate experience.

We reviewed Montana in State Group C, which includes Alabama, Alaska, Arizona, Louisiana, Mississippi, Montana, Nevada, Washington, West Virginia and Wyoming.

Overall and by-table indicated changes are calculated using state group weights.

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EFFECT ON MANUAL PAGES	Upon implementation of this filing, which revises Premises/Operations increased limit factors, we will publish revised manual pages in Division Six of the Commercial Lines Manual. The revised increased limit factors will appear in Rule 56 as Tables 56.B.1., 56.B.2. and 56.B.3.
COMPANY DECISION	We encourage each insurer to decide independently whether the judgments made and the procedures or data used by ISO in developing increased limit factors are appropriate. We have included within this document the information upon which ISO relied in order to enable companies to make such independent judgments. The data underlying the enclosed material comes from companies reporting to ISO. Therefore, the ISO statistical database is much larger than any individual company's. A broader database enhances the validity of the ratemaking analysis. At the same time, an individual company may benefit from a comparison of its own experience to the aggregate ISO experience and may reach valid conclusions with respect to the manner in which its own costs can be expected to differ from ISO's projections based on the aggregate data. Some calculations included in this document involve areas of ISO staff judgment. Each company should carefully review and evaluate its own experience in order to determine whether the increased limit factors developed by ISO are appropriate for its use. This material has been developed exclusively by the staff of ISO.

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SUMMARY OF INCREASED LIMIT FACTOR CHANGES

Exhibit 1 (*Summary of Increased Limit Factor Changes*) provides a summary of the current, indicated and selected per occurrence increased limit factors for Premises/Operations.

SUMMARY OF REVISED INCREASED LIMIT FACTORS

Exhibit MP (*Manual Pages*) displays the revised Premises/Operations increased limit factors as they will appear in Division Six of the Commercial Lines Manual for Tables 1, 2 and 3 (corresponding to Tables 56.B.1., 56.B.2. and 56.B.3. in the manual rule pages, respectively).

The increased limit factors shown are the ratio of the sum of indemnity, allocated loss adjustment expense, unallocated loss adjustment expense and risk load at each specific limit to the same sum evaluated at the basic limit of $100,000 per occurrence/$200,000 aggregate. Therefore, the factor listed for the basic limit is 1.00.

Certain factors have been judgmentally modified to maintain consistency within the tables. This ensures that the relative incremental costs (as measured by the change in ILFs divided by change in policy limits) for progressively higher occurrence and/or aggregate limits do not increase (i.e., the marginal costs are either constant or decreasing).

Exhibit 2 (*Comparison of Current and Revised Occurrence/Aggregate Increased Limit Factors*) compares the current and revised occurrence/aggregate increased limit factors for a sample of policy limits for Premises/Operations.

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SCOPE OF REVISION

OCCURRENCE/ AGGREGATE SIMULATION

To generate the occurrence/aggregate increased limit factors, we begin with the calculation of indicated increased limit factors, displayed in **Exhibits 3-5**. We reflect the aggregate policy limit by combining the indemnity severity distribution (determined from the parameters provided in **Exhibit 8**) to model the loss size, and a weighted mixture of negative binomial distributions (discussed below) to model the number of occurrences per policy. We use the frequency model to simulate occurrence counts (for a large number of simulated policies), and the severity model to generate the losses for the simulated occurrences. This combined distribution produces limited losses at various combinations of occurrence and aggregate limits.

The probability of k occurrences is equal to

$$p_k = \sum_j w_j p_{kj}$$

where:
w_j is the weight of each component negative binomial distribution j;
and p_{kj} is the probability of k occurrences for each component distribution, such that:

$$p_{kj} = \frac{\Gamma(k + r_j)}{k!\ \Gamma(r_j)} \left(\frac{\beta_j}{1 + \beta_j}\right)^{r_j} \left(\frac{1}{(1 + \beta_j)^k}\right)$$

The grand mean of the mixture distribution is equal to:

$$m = \sum_j w_j m_j$$

where m_j is the mean for component distribution j, calculated as:

$$m_j = \frac{r_j}{\beta_j}$$

Exhibit 13 (*Mixed Negative Binomial Frequency Parameters*) shows the frequency parameters for Premises/Operations determined on a multistate basis.

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SUPPORTING MATERIAL

OVERVIEW OF INCREASED LIMIT FACTOR CALCULATIONS

This section describes the methods we use to calculate increased limit factors for policies that are subject to occurrence limits, but not annual aggregate limits. Section A describes the aggregate model by which we determine our occurrence/aggregate increased limit factors. The per-occurrence loss distributions and loss adjustment expense provisions that are described in this section are key components of this aggregate model. Also, the calculation of increased limit factors for occurrence-only limits illustrates the principles underlying the calculation for occurrence/aggregate limits.

ISO defines an increased limit factor (ILF) as the ratio of the expected cost (to the insurer) of a higher limit policy divided by the expected cost of a basic limit policy. The cost components of the occurrence-limit increased limit factor calculation are:

- Limited Average Severity (LAS)

  The average indemnity per occurrence, limited to a given policy limit, at ultimate settlement value, and reflecting trend to the average accident date in the prospective experience period.

  In this document, we use the term "indemnity" to mean the amount paid to the claimant (excluding all loss adjustment expense). Indemnity is subject to policy limits. We construct an occurrence-size distribution that describes the indemnity before the effect of policy limits. By using this distribution, we can calculate expected future indemnity for any given policy limit.

- Allocated Loss Adjustment Expense (ALAE)

  The average claim settlement expense per occurrence for those expenses in the settlement process that can be assigned to an individual claim. The largest component of ALAE is legal defense costs.

- Unallocated Loss Adjustment Expense (ULAE)

  The average claim settlement expense per occurrence for those expenses in the settlement process that cannot be assigned to an individual claim (e.g., the salaries of claims adjusters).

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OVERVIEW OF INCREASED LIMIT FACTOR CALCULATIONS (continued)

- Risk Load (RL)

  A loading that varies by policy limit and reflects the greater risk of issuing higher limit policies, with the fundamental purpose of making each policy limit being written equally attractive to insurers. The ISO risk load model accomplishes this by offsetting the greater risk associated with higher limit policies with an appropriate risk load provision that increases as the policy limit increases. The procedure recognizes two kinds of risk:

  Process Risk - the inherent variability of the insurance process, reflected in the difference between actual losses and expected losses.

  Parameter Risk - the inherent variability of the estimation process, reflected in the difference between theoretical (true but unknown) expected losses and the estimated expected losses.

The ISO increased limit factor is the ratio of these costs at a specified limit divided by the corresponding costs at the basic limit. Given a basic limit *b*, the factor at occurrence policy limit *PL* is as follows:

$$ILF(PL) = \left[\frac{LAS(PL) + ALAE(PL) + ULAE(PL) + RL(PL)}{LAS(b) + ALAE(b) + ULAE(b) + RL(b)} \right]$$

Exhibits 3 through **5** *(Calculation of Increased Limit Factors)* show the indicated and selected occurrence-limit increased limit factors for each of the increased limits tables from ISO's 2019 General Liability Premises/Operations increased limits review. Also shown are the underlying components of the calculation by limit. An overview of these four components of the occurrence-limit increased limit factor calculation follows.

STATE GROUPS

For Premises/Operations, we review the data by state or state group. Only the largest states have sufficient volume to review individually. The largest 15 states are reviewed individually. The remaining 37 jurisdictions are grouped into a three-tiered state group structure to accommodate relatively low, medium and high ILF state groups - State Groups A, B and C. State Group A is comprised of the lowest ILF jurisdictions; State Group C is comprised of the highest ILF jurisdictions; and State Group B contains the remainder of the jurisdictions.

To generate the complements of credibility, we group each of the individually reviewed states with either State Group A, B or C, creating three larger state group complements encompassing all states. State group experience is combined with the corresponding state group complement experience at each layer of loss to enhance the stability of the increased limit factors. This is an application of the standard actuarial practice of credibility weighting. We provide a definition of the state group complements (referred to as A', B' and C') and discuss credibility weighting in more detail in the Combining State Group Data with State Group Complement Data subsection later in this document.

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STATE GROUPS (continued)

For Premises/Operations, this state is reviewed in State Group C, which includes Alabama, Alaska, Arizona, Louisiana, Mississippi, Montana, Nevada, Washington, West Virginia and Wyoming.

Overall and by-table indicated changes are calculated using state group weights. We use multistate (all state groups) experience for the following calculations:

- unallocated loss adjustment expense, and
- severity trend.

DATA FOR INDEMNITY ANALYSIS

The limited average severity in this increased limits review is modeled using loss data reported to ISO under the Commercial Statistical Plan via prior ("pre-CGL") and current ("CGL") applicable subline codes. We also include excess and umbrella data reported under the Commercial Statistical Plan, to add greater credibility to higher layer analysis. We include additional data from the ISO Annual Call for Excess and Umbrella Policy Claims. This data enhances the credibility of our ILFs in the highest layers of loss that we evaluate.

The data is comprised of paid (settled) occurrences on occurrence coverage policies with accident dates between January 1, 2004 and December 31, 2017, and average payment dates between January 1, 2013 and December 31, 2017. The data is evaluated as of March 31, 2018.

We consider an occurrence to be settled if it has no outstanding reserve. If there are multiple payments, we consider the average payment date to be the dollar-weighted average of the dates of the individual payments.

We use "payment lag" or "lag" to measure the amount of time between the occurrence and the payments made towards the loss settlement. A lag of 1 indicates that the average payment date is in the same accident year as the occurrence. A lag of 2 indicates that the average payment date falls in the following year, and so on.

For each occurrence we determine the severity table, accident year, payment lag, indemnity amount, policy limit, and any applicable deductible or attachment point.

COMPOSITE-RATED RISKS

Insurers report composite-rated risk (CRR) data to ISO without detailed classification information. However, since a significant portion of our data is composite-rated and using it also would enhance credibility, we traditionally have employed an allocation approach to include CRR data in our calculation of increased limit factors by table.

COMPOSITE-RATED RISKS (continued)

Starting with our 2019 review, we are assigning CGL CRR data to tables outright as with experience from typically mapped classes, based on empirical severity analysis performed during the review. The new Premises/Operations CRR table assignments are as follows:

Table	CRR Classifications
1	40050, 52050, 52350, 52450, 52950, 70350, 70650, 71150, 80050, 80150
2	12950, 15150, 20150, 20250, 20350, 49950, 50050, 60050, 70050, 70250, 70450, 70550, 94050, 98050, 98550
3	01050, 10050, 12150, 12250, 15050, 15250, 15350, 20050, 20450, 20550, 48050, 49050, 52250, 93050, 98750

We continue to allocate pre-CGL CRR data to the individual tables as in past reviews: using the accident year, payment lag and indemnity amount of a given pre-CGL CRR occurrence, we can make a Bayesian estimate of the probability it belongs in each table based on its known characteristics.

We then allocate part of each such occurrence to the various tables using this Bayesian analysis. Thus, we might consider a single $100,000 occurrence to be 1/3 of a “Table 1” occurrence, 1/2 of a “Table 2” occurrence, and 1/6 of a “Table 3” occurrence. In each case, the amount of the (fractional) occurrence would remain $100,000. We describe this process further in the Bayesian-related sections later in this document.

EXCESS AND UMBRELLA DATA

As stated, along with the umbrella and excess data reported to ISO under the Commercial Statistical Plan, we include additional data from the ISO Annual Call for Excess and Umbrella Policy Claims. This data enhances the credibility of our increased limit factors but does not affect the lowest layers.

These excess and umbrella policies have attachment points that exclude smaller losses much the same way as a large deductible would. While we can reconstruct the full size of loss for those occurrences greater than the attachment point of their policy, occurrences below the attachment point are not reported.

When we construct the empirical survival distribution, we exclude occurrences where the attachment points do not meet certain criteria, to avoid bias. We describe this in more detail later in this document. Also, because excess and umbrella data is not reported in class detail, we allocate the data to each table using the same Bayesian procedure that we apply for pre-CGL CRR data.

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SUPPORTING MATERIAL

MIXED EXPONENTIAL MODEL

For each table, we fit a continuous distribution to the lag-weighted occurrence-size distribution from the data. The resulting distribution produces the limited average severity component of the increased limit factor.

Using a continuous distribution (such as the mixed exponential) offers several advantages over using a purely empirical fit, including:

- calculation of limited average severity for all possible limits,
- smoothing of data,
- simplified handling of trend, and
- calculation of higher moments used in risk load.

The fitting procedure uses a mixture of exponential distributions to model indemnity. ISO found that the mixed exponential distribution provides a good fit to empirical data over a wide range of loss sizes, is flexible and is simple to use.

OVERVIEW OF MIXED EXPONENTIAL PROCESS

The major steps in the calculation of the limited average severities of the indemnity are:

1. Trend
Trending the indemnity amount of each occurrence to reflect the expected conditions during the period when the increased limit factors are assumed to be in effect.

2. Construction of the Empirical Survival Distributions
Using the trended data to calculate the empirical survival distributions by payment lag for each table and state group.

3. Payment Lag Process
Combining the empirical distributions for each payment lag to produce an overall empirical survival distribution for each table and state group.

4. Tail of the Distribution
Smoothing the tail of the lag-weighted empirical survival distribution for each table separately for each of the larger state group complements for Premises/Operations.

5. Combining State Group data with State Group Complement data
Credibility-weighting the Premises/Operations state group experience with the experience of the corresponding state group complement.

6. Fitting a Mixed Exponential Distribution
Fitting a mixed exponential model to the empirical survival distribution.

7. Final Limited Average Severities
Using the fitted mixed exponential distribution to generate limited average severities.

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INDEMNITY SEVERITY TREND

For a given payment lag, we expect severity to increase by the inflation rate from accident year to accident year.

If annual inflation is 4.0%, an injury that resulted in a $100,000 paid claim in 2018 should cost 1.04 x $100,000 in 2019. The probability of that particular accident stays the same – only the nominal value of it changes.

To bring different accident years to the same level, we project each occurrence from the average date of its accident year to December 1, 2020, one year beyond the assumed effective date of December 1, 2019. In this filing, we select an annual trend of +6.0% for Premises/Operations. This compares to a trend of +5.5% in the most recently filed 2018 increased limits review.

We selected the annual severity trend factor based on the data from the underlying paid loss development triangles from this increased limits review. Trend indications are currently reviewed on a multistate basis. Manually-rated classes and A-rated classes as well as CRR classes are included in the increased limits development triangles for all significant types of loss related to General Liability.

Exhibit 6 (*Indemnity Severity Trend Selection*) provides the annual paid basic limit and total limits severity trend indications. We also provide a measure of the goodness-of-fit statistic for each of the various multi-year trend fits.

CONSTRUCTION OF THE EMPIRICAL SURVIVAL DISTRIBUTIONS

The construction of the empirical survival distributions is based on the Product-Limit Estimator described in Loss Models: From Data to Decisions[1]. First, paid (settled) occurrences are organized by accident year and payment lag and trended to the average accident date for which the loss distribution is desired.

Payment lags seven and beyond generally have similar loss sizes and are combined to increase credibility. Other lags are handled individually. We further define payment lag and explain the reasons for its use later in the explanatory materials.

Next, a survival distribution is constructed for each payment lag using discrete loss size layers. The probability that an occurrence exceeds the upper bound of a discrete layer given that it exceeds the lower bound of the layer is known as the conditional survival probability (CSP). The ground-up survival distribution is generated by multiplying the successive CSPs of the discrete layers.

[1] S. A. Klugman, H.H. Panjer, and G. E. Willmot, *Loss Models: From Data to Decisions*, John Wiley and Sons, New York, 2004

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CONSTRUCTION OF THE EMPIRICAL SURVIVAL DISTRIBUTIONS (continued)

This procedure allows for the easy inclusion of censored losses as well as excess, umbrella and deductible data. Two conditions must be met for an occurrence to be used in the calculation of the conditional survival probability in a particular layer of loss. These conditions are:

- The policy limit (plus attachment point or deductible) must be greater than or equal to the upper bound of the layer of loss. This avoids a downward severity bias by excluding losses that are precluded by their policy limit from penetrating the upper bound of a layer of loss.

- Only those occurrences with attachment points or deductibles less than or equal to the lower bound of the layer of loss are included. This condition is necessary to avoid an upward severity bias since loss information below the attachment point or deductible is unknown.

ILLUSTRATION

An illustration should aid in the conceptual understanding of this construction.

Assume we have twelve occurrences, all for a single payment lag. We will calculate the empirical survival probabilities for three layers using combinations of conditional survival probabilities. The three layers used are $10,000, $20,000 and $40,000 (in practice we begin with layers as small as $10, but larger layers better illustrate the handling of deductibles and policy limits). The following two pages display sample calculations for these three layers.

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Illustrative Data (Trended) for One Payment Lag

Occurrence ID Number	Occurrence Size	Attachment Point	Policy Limit	Comment
1	5,000	0	15,000	
2	5,000	0	15,000	
3	15,000	0	15,000	Censored Data
4	5,000	7,500	15,000	Deductible Data
5	5,000	0	30,000	
6	15,000	0	30,000	
7	25,000	0	30,000	
8	10,000	15,000	30,000	Excess Data
9	15,000	0	100,000	
10	25,000	0	100,000	
11	30,000	0	100,000	
12	50,000	15,000	100,000	Excess Data

Where attachment point is non-zero, we define policy limit as the maximum payment.

Conditional Survival Probabilities

	Condition:
$CSP_{e1}(10{,}000 \mid 0) =$ $P(X \geq 10{,}000 \mid X > 0)$	$PL + AP \geq 10{,}000$ $AP = 0$
$CSP_{e1}(20{,}000 \mid 10{,}000) =$ $P(X \geq 20{,}000 \mid X \geq 10{,}000)$	$PL + AP \geq 20{,}000$ $AP \leq 10{,}000$
$CSP_{e1}(40{,}000 \mid 20{,}000)$ $P(X \geq 40{,}000 \mid X \geq 20{,}000)$	$PL + AP \geq 40{,}000$ $AP \leq 20{,}000$

where AP = attachment point, PL = policy limit, X= loss size, e_1 = empirical lag 1

Calculation of Conditional Survival Probability at $10,000

$CSP_{e1}(10{,}000 \mid 0) = P(X \geq 10{,}000 \mid X > 0) =$ number of occurrences with:
occurrence size + AP $\geq$ 10,000,
policy limit + AP $\geq$ 10,000, and AP = 0
number of occurrences with:
occurrence size + AP > 0,
policy limit + AP $\geq$ 10,000, and AP = 0

= 6 (occurrences 3, 6, 7, 9, 10, 11)
= 9 (occurrences 1, 2, 3, 5, 6, 7, 9, 10, 11)

Only occurrences with policy limit plus attachment point greater than or equal to 10,000 are used. Only occurrences with attachment point equal to zero are used.

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Calculation of Conditional Survival Probability at $20,000

$$CSP_{e1}(20{,}000 \mid 10{,}000) = P(X \geq 20{,}000 \mid X \geq 10{,}000) = \frac{\text{number of occurrences with: occurrence size + AP} \geq 20{,}000\text{, policy limit + AP} \geq 20{,}000\text{, and AP} \leq 10{,}000}{\text{number of occurrences with: occurrence size + AP} \geq 10{,}000\text{, policy limit + AP} \geq 20{,}000\text{, and AP} \leq 10{,}000}$$

$$= \frac{3 \text{ (occurrences 7, 10, 11)}}{6 \text{ (occurrences 4, 6, 7, 9, 10, 11)}}$$

Only occurrences with policy limit plus attachment point greater than or equal to 20,000 are used. Only occurrences with attachment point less than or equal to 10,000 are used.

Calculation of Conditional Survival Probability at $40,000

$$CSP_{e1}(40{,}000 \mid 20{,}000) = P(X \geq 40{,}000 \mid X \geq 20{,}000) = \frac{\text{number of occurrences with: occurrence size + AP} \geq 40{,}000\text{, policy limit + AP} \geq 40{,}000\text{, and AP} \leq 20{,}000}{\text{number of occurrences with: occurrence size + AP} \geq 20{,}000\text{, policy limit + AP} \geq 40{,}000\text{, and AP} \leq 20{,}000}$$

$$= \frac{1 \text{ (occurrence 12)}}{4 \text{ (occurrences 8, 10, 11, 12)}}$$

Only occurrences with policy limit plus attachment point greater than or equal to 40,000 are used. Only occurrences with attachment point less than or equal to 20,000 are used.

Calculation of Empirical Survival Distribution

The CSPs generate the following empirical survival probabilities:

$$\begin{aligned} S_{e1}(10{,}000) &= P(X \geq 10{,}000) = CSP_{e1}(10{,}000 \mid 0) = P(X \geq 10{,}000 \mid X > 0) \\ &= 6/9 \end{aligned}$$

$$\begin{aligned} S_{e1}(20{,}000) &= P(X \geq 20{,}000) = CSP_{e1}(10{,}000 \mid 0) * CSP_{e1}(20{,}000 \mid 10{,}000) \\ &= P(X \geq 10{,}000 \mid X > 0) * P(X \geq 20{,}000 \mid X \geq 10{,}000) \\ &= 6/9 * 3/6 = 1/3 \end{aligned}$$

$$\begin{aligned} S_{e1}(40{,}000) &= P(X \geq 40{,}000) = CSP_{e1}(10{,}000 \mid 0) * CSP_{e1}(20{,}000 \mid 10{,}000) * CSP_{e1}(40{,}000 \mid 20{,}000) \\ &= P(X \geq 10{,}000 \mid X > 0) * P(X \geq 20{,}000 \mid X \geq 10{,}000) * P(X \geq 40{,}000 \mid X \geq 20{,}000) \\ &= 6/9 * 3/6 * 1/4 = 1/12 \end{aligned}$$

In practice, to generate the trended empirical loss distribution for each lag, we use sixty-eight discrete loss size layers to allow for a refined selection of the tail-smoothing parameters, discussed in the Tail of the Distribution section.

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PAYMENT LAG PROCESS

Development for paid (settled) data has two aspects. One aspect is that many occurrences are paid within a short period of time after the accident, with a small number taking longer – sometimes much longer – to be paid. The second aspect is the tendency of larger occurrences to take longer to be paid.

To properly model an accident year at ultimate, we must include each payment lag with its appropriate weight. We do this by:

- accounting for the rate of payment using the probability-of-payment-lag model, and
- constructing severity distributions by payment lag.

A "lag weighting" procedure then combines the by-lag empirical loss distributions to generate an overall distribution. This procedure implicitly accounts for development as all possible payment lags are represented and given weight at the prospective average accident date. We refer to the distribution of the overall survival probabilities by size of loss as the "empirical survival distribution function (SDF)".

PAYMENT LAG

Payment lag is the length of time between when an accident occurs and the date when the associated indemnity is paid. In the mixed exponential model, the payment date is the dollar-weighted average of the dates of the indemnity payments. ISO calculates payment lag based on the year in which an accident occurs and the year in which the occurrence is paid:

Payment Lag = (Payment Year - Accident Year) + 1

Payment lag can vary considerably by line of business and by type of claim. While most property claims are paid quickly, liability claims generally take longer to settle, particularly those involving protracted litigation. Among liability claims, there is considerable variation in payment lag.

DIFFERENCES IN LOSS SIZES BY PAYMENT LAG

Generally, occurrences with longer payment lags involve higher loss sizes. For example, the average loss size for occurrences paid in lag 4 will tend to be considerably higher than the average loss size for those paid in lag 1.

The Mixed Exponential Methodology reflects this by fitting (the continuous mixed exponential distribution) to a lag-weighted empirical survival distribution. We do not directly fit to the severity distributions of individual lags.

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PAYMENT LAG DISTRIBUTION

The payment lag distribution is modeled to avoid distortions that may otherwise result from:

- differing exposure amounts by accident year,
- an asymmetrical experience period with fewer than five accident years for lags eleven through fourteen, and
- a finite number of lags (no data for lags beyond fourteen).

The lag-weighting procedure implicitly accounts for ultimate development, as all possible payment lags are represented and given weight at the prospective average accident date.

The payment lag model uses three parameters (R1, R2 and R3) to generate the weights given to the severity distribution associated with each payment lag. The parameters can be represented as follows:

$$R1 = \frac{\text{expected percentage of occurrences paid in lag 2}}{\text{expected percentage of occurrences paid in lag 1}}$$

$$R2 = \frac{\text{expected percentage of occurrences paid in lag 3}}{\text{expected percentage of occurrences paid in lag 2}}$$

$$R3 = \frac{\text{expected percentage of occurrences paid in lag (n+1)}}{\text{expected percentage of occurrences paid in lag (n)}}, \text{ for all } n \geq 3$$

The weights for each lag are then determined as follows:

lag 1 weight = $1 / k$, where $k = \{1 + R1 + [R1 \bullet R2] / [1-R3]\}$
lag 2 weight = $R1 / k$
lag 3 weight = $R1 \bullet R2 / k$
lag 4 weight = $R1 \bullet R2 \bullet R3 / k$
lag 5 weight = $R1 \bullet R2 \bullet R3^2 / k$
lag 6 weight = $R1 \bullet R2 \bullet R3^3 / k$
lag 7 weight = $R1 \bullet R2 \bullet [R3^4 / (1-R3)] / k$,

Note that the lag 7 weight includes lag 7 and all subsequent lags.

The lag weights represent the percentage of ground-up occurrences in each lag. Therefore, occurrences from deductible, umbrella or excess policies with non-zero attachment points are not included.

METHOD OF ESTIMATION: PAYMENT LAG PARAMETERS

For stability, we calculate the payment lag parameters (R1, R2 and R3) via maximum likelihood. Except for pre-CGL CRR data, an occurrence with accident year *a* and payment lag *l* is reflected in the likelihood function by the probability that the lag equals *l* given that the accident year equals *a*. This conditional probability can be easily expressed in terms of the payment lag parameters.

METHOD OF ESTIMATION: PAYMENT LAG PARAMETERS (continued)

For a pre-CGL CRR occurrence, the probability that the loss comes from a given table is computed by the procedure described later in the Bayesian-related sections. Each pre-CGL CRR occurrence generates several probabilities, one for each table. These probabilities are treated as fractional occurrences in the likelihood function.

Exhibit 7 *(Payment Lag Parameters and Lag Weights)* shows the resulting values of these parameters.

TAIL OF THE DISTRIBUTION

For the higher limits of liability, experience may be sparse in the tail of the distribution. To account for this, and to limit random fluctuations in the higher limits between consecutive reviews, we implicitly smooth the tails of the empirical state group distributions by smoothing the tails of the larger state group complement distributions (referred to as A', B' and C'). We select truncation points above which the state group complements' empirical survival distribution functions can be relatively less stable. The truncation points in this filing are:

Line/State Group	Table 1	Table 2	Table 3
PremOps C'	2,500,000	2,500,000	1,400,000

Then we select a parametric curve family that successfully models the behavior of the empirical distributions in the layers around the truncation point. During this process, we examine which curve parameters would minimize the overall severity difference between the empirical and smoothed distributions. The resulting curve is used to extrapolate the empirical distributions above the truncation point. The state group complements' empirical distributions below the truncation point are unaffected by this procedure.

This procedure smooths the tail of the state group complements' empirical distributions by extending relationships from the highest credible limits (those limits around the truncation point) to those limits above the truncation point. For each state group, we use the shape of the appropriate extrapolated larger state group complement distribution to extend the credibility-weighted state group distribution above the truncation point. Essentially, this smooths the tail of the distribution for each state group and table. We then fit a mixed exponential distribution to the resulting SDF for each increased limits table.

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COMBINING STATE GROUP DATA WITH STATE GROUP COMPLEMENT DATA

For Premises/Operations, we construct the empirical survival distribution by state or state group for each table. State or state group conditional survival probabilities (CSPs) are weighted with the larger, more representative state group complements' CSPs at each layer. Grouping states or state groups with larger state groupings of similar experience produces more consistent and intuitive complements of credibility. To generate the complements of credibility, we grouped each of the individually reviewed states with either State Group A, B or C, creating three larger state group complements. The sum of these larger state group complements by definition includes all multistate data.

The definitions of the state group complements (referred to as A', B', and C') are as follows:

- A': State Group A, NC, OH, VA, WI
- B': State Group B, FL, GA, IN, MA, MI, NJ, PA, TX
- C': State Group C, CA, IL, NY

The weight assigned to each state group's CSP in each layer is an increasing function of the number of occurrences for that state group in that layer. Thus, greater weight is given to state group experience in lower layers where greater volume contributes to stability for experience by state group.

The formula used is:

Weighted $CSP_i = (Z_i) \times$ State Group $CSP_i + (1 - Z_i) \times$ State Group Complement CSP_i, where:

- $Z_i = N_i / (N_i + K)$,
- i is the i^{th} loss size layer, and
- N_i is the number of occurrences that can be used to evaluate CSP_i for the state group, and K=300 for state group complement A', 200 for state group complement B', and 100 for state group complement C'.

The values of K were selected based on an evaluation of the total variability of CSPs by layer compared to the variability across all state groups within the state group complement. This is an application of Bühlmann-Straub credibility procedures to CSPs. Bühlmann-Straub credibility procedures are described in a number of actuarial texts, including Loss Models: From Data to Decisions[3].

As stated in the Tail of the Distribution section, for the highest layers of loss, we first extrapolate the CSPs for the three larger state group complements A', B' and C' through the tail smoothing process.

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[3] S. A. Klugman, H.H. Panjer, and G. E. Willmot, *Loss Models: From Data to Decisions*, John Wiley and Sons, New York, 2004

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FITTING A MIXED EXPONENTIAL DISTRIBUTION

ISO models the lag-weighted empirical survival distribution function for each table with the best fitting mixed exponential distribution. The lag-weighted SDFs reflect smoothing and, if applicable, credibility weighting. The resulting mixed exponential distribution produces the limited average severity component of the increased limit factor.

THE SIMPLE EXPONENTIAL DISTRIBUTION

To understand the mixed exponential distribution, first consider the simple exponential distribution. The simple exponential is a one-parameter distribution. The formulas for the survival distribution function (SDF(x)) and the limited average severity (LAS) at a given policy limit (PL) for an exponential distribution with mean parameter μ are given by:

$$\mathrm{SDF}(x) = e^{-\left(\frac{x}{\mu}\right)} = 1 - \mathrm{CDF}(x)$$

$$\mathrm{LAS}(PL) = \mu\left[1 - e^{-\left(\frac{PL}{\mu}\right)}\right]$$

THE MIXED EXPONENTIAL DISTRIBUTION

The mixed exponential distribution is a weighted average of exponential distributions. Each exponential distribution has two parameters, a mean μ_i and a weight w_i. Note that the SDF at zero is unity, and the weights sum to 1.000000.

The formulas for the survival distribution function and limited average severity for the mixed exponential distribution are the weighted averages of the respective single exponential formulas:

$$\mathrm{SDF}(x) = \sum_i \left[w_i e^{-\left(\frac{x}{\mu_i}\right)}\right]$$

$$\mathrm{LAS}(PL) = \sum_i w_i \mu_i \left[1 - e^{-\left(\frac{PL}{\mu_i}\right)}\right]$$

ISO found that the mixed exponential distribution is flexible and simple to use and provides a good fit to empirical data over a wide range of loss sizes. In fact, any distribution whose probability density function (pdf) has alternating derivatives:

$$\begin{aligned} \mathrm{pdf}(x) &> 0, \\ d\,\mathrm{pdf}(x)/dx &< 0, \\ d^2\,\mathrm{pdf}(x)/dx^2 &> 0, \\ d^3\,\mathrm{pdf}(x)/dx^3 &< 0, \quad \text{etc., for all } x > 0, \end{aligned}$$

can be constructed as a mixture of exponentials with positive means and weights. Such distributions (including the mixed Pareto, if it has a finite mean) can be thought of as special cases of the mixed exponential distribution.

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THE MIXED EXPONENTIAL DISTRIBUTION SEVERITY PARAMETERS

ISO estimates the mixed exponential distribution parameters using minimum distance estimation. We compare the model SDF to the empirical SDF at each of the discrete loss size layers resulting from the construction.

We seek a mixed exponential distribution that minimizes the weighted sum of the square of the differences of these survival probabilities (model minus empirical) taken at each loss size layer. This procedure is known as the “minimum distance” method.

The number of exponential distributions needed to produce an optimal fit to the empirical SDF may vary by table and can be as large as necessary.

For General Liability, we allow means up to $100 million, to follow the smoothed empirical distribution in layers above $10 million more closely. Allowing means up to $100 million tends to increase the number of means (and weights) for the fitted distribution in a given table, while having minimal effect on limits up to $10 million, the highest limit for which we publish increased limit factor information.

Exhibit 8 (*Parameters for Mixed Exponential Distributions)* displays the mixed exponential parameters (means and weights) for each increased limits table.

MAY NOT BE APPLICABLE FOR ALL POLICY LIMITS

ISO’s standard increased limits tables (shown in **Exhibits 3** through **5**) provide increased limit factors up to the $10,000,000 per occurrence policy limit. **We encourage the use of supplemental sources of information for analysis of layers above $10,000,000.**

FINAL LIMITED AVERAGE SEVERITIES

ISO calculates the limited average severities using the fitted mixed exponential distributions for each table. The *Mixed Exponential Distribution* section gives the formula for the limited average severity of a mixed exponential distribution. **Exhibit 8** (*Parameters for Mixed Exponential Distributions)* shows the individual by-table severity parameters used in this formula for each increased limits table.

Exhibit 9 (*Comparison of Limited Average Severities)* compares the fitted limited average severities to the empirical limited average severities. The empirical limited average severities are constructed in a manner analogous to the empirical survival distributions. The same conditions and assumptions are used in combination with actual trended loss amounts in each layer.

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BAYESIAN ANALYSIS

As stated, we utilize a Bayesian approach to allocate pre-CGL CRR, excess and umbrella occurrences to each increased limits table. For each payment lag, the Bayesian analysis is as follows:

$$\text{P(Table | Indemnity} = \frac{\text{P(Indemnity} \mid \text{Table)} \times \text{P(Table)}}{\sum \text{P(Indemnity} \mid \text{Table)} \times \text{P(Table)}}$$

The sum in the denominator is over all tables.

Here P(Table | Indemnity) is the conditional probability (within the payment lag) that an occurrence comes from the specified table, given the indemnity amount.

P(Table) is the marginal probability (within the payment lag) that an occurrence comes from the specified table.

Clearly, the table probabilities sum to one:

$$\sum \text{P(Table} \mid \text{Indemnity)} = 1;$$

that is, 100% of each occurrence is allocated.

We estimate P(Table) as the ratio of two sums:

$$\text{P(Table)} = \frac{\text{\# of occurrences with known table in this table}}{\text{\# of occurrences with known table in all tables}}$$

Here we restrict both the numerator and denominator to the payment lag under consideration.

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BAYESIAN ALLOCATION AND EMPIRICAL SURVIVAL DISTRIBUTIONS	For an occurrence with unknown table not censored by policy limits, we use: P(Indemnity | Table) = f(Indemnity Layer), where f(Indemnity Layer) is the empirical probability of an occurrence being in the indemnity layer. This empirical probability is the difference of the empirical SDF (for the table-payment lag combination) between the top and the bottom of the layer. For an occurrence with unknown table censored by policy limits, we use: P(Indemnity | Table) = SDF(Indemnity Layer), where SDF(Indemnity Layer) is the empirical SDF evaluated at the bottom of a layer, for the table-payment lag combination.
ALLOCATED DATA IN PROBABILITY-OF-PAYMENT-LAG MODEL	We allocate pre-CGL CRR data to tables within an accident year and payment lag using the Bayesian analysis described in previous section. We then have revised occurrence counts by accident year, payment lag, and table. These counts include fractional occurrences from the pre-CGL CRR data. These counts are the raw data for our probability-of-payment-lag model. We do not include excess and umbrella data, or deductible data, in the probability-of-payment-lag model. This avoids bias from not including unreported occurrences smaller than the policy attachment points or deductibles.

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ALLOCATED LOSS ADJUSTMENT EXPENSE

The standard liability policy contains a policy limit which represents the maximum amount an insurer will pay for any loss for which the insured is liable. However, the limit does not apply to the loss adjustment expenses. For this reason, we estimate ALAE per occurrence as a single amount that does not vary by policy limit.

For each table, we estimate allocated loss adjustment expense (ALAE) per occurrence as the product of two numbers. The first number is the ratio of paid ALAE to paid total limits (all limits combined) indemnity. The second number is the average (across all policy limits) limited average severity calculated from the mixed exponential model.

To calculate the ALAE per occurrence, we first calculate the ratio of dollars of ALAE to dollars of total limits indemnity for the seven next-to-latest available accident years (the latest accident year is excluded from the average because its development tends to be less stable). We develop these ratios to ultimate maturity.

To further enhance stability, we use a best 5-of-7 criterion and eliminate the lowest and highest paid ratios. We then average the best 5-of-7 paid ratios to determine the overall ALAE to total limits indemnity ratio for each table.

The fitted total limits average severity for each table is a weighted average of the limited average severities at the different policy limits. The weights used are occurrences from the second, third and fourth latest accident years.

For each table, the multi-year average ALAE to total limits indemnity ratio is then multiplied by the final fitted total limits average severity to calculate the ALAE per occurrence provision for use in computing increased limit factors. The total limits average severity reflects trend to the average prospective accident date. This effectively contemplates trend in ALAE in a more stable manner than relying on a separate trend analysis of ALAE.

Exhibit 10 *(Calculation of Allocated Loss Adjustment Expense per Occurrence)* shows the calculation of the allocated loss adjustment expense component for Premises/Operations Liability.

UNALLOCATED LOSS ADJUSTMENT EXPENSE

We calculate the unallocated loss adjustment expense at each limit of liability as a percentage of the sum of the limited average severity and the ALAE at that liability limit. For this filing, we select the ULAE load of 8.5% based on a five-year average of multistate financial data reported to ISO.

Exhibit 11 (*Development of Unallocated Loss Adjustment Expense Factor*) shows the derivation of this factor.

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RISK LOAD

Our increased limits methodology incorporates a procedure to reflect the relatively higher risk or variation in experience associated with higher limit policies. The model that we use, the Competitive Market Equilibrium Risk Load Model[1], assumes that the insurance marketplace is competitive and efficient. In a competitive marketplace, individual insurers cannot influence the marketplace price. While individual insurers cannot influence the risk associated with a given policy limit, they will attempt to maximize their expected net revenue by choosing which lines and policy limits to write. This assumption is consistent with rational economic behavior and is reinforced by solvency regulation.

In an efficient marketplace, the supply of insurance matches the demand. ISO uses the distribution of basic limit losses by policy limit to represent the market demand for insurance at each limit. The model determines a set of risk loads that match supply and demand at each policy limit.

The variability of losses is caused by process risk and parameter risk:

- Process risk reflects the inherent uncertainty of the insurance process. Even if one could estimate expected losses exactly, actual losses will almost certainly differ from the expected. We derive the process risk component from the parameters of the indemnity severity distribution.

- Parameter risk reflects the risk of not estimating expected losses accurately. The derivation of the parameter risk component is based on the historical variation of losses.

These two risk elements combined comprise the total risk load at each policy limit.

ISO's risk load formulas use a parameter, lambda (λ), which governs the total amount of risk load over all policy limits for (non-professional) commercial liability tables. We determine lambda so that the ratio of the average indicated increased limit factor with risk load to the average indicated increased limit factor without risk load is equal to 1.06 for all General and Commercial Automobile Liability tables combined. For this state group, this ratio is 1.056 for Premises/Operations Liability.

Exhibit 12 *(Risk Load Parameters)* shows parameters used in the calculation of risk load.

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[1] G. G. Meyers, *Competitive Market Equilibrium Risk Load Model for Increased Limits Ratemaking*, Proceedings of the Casualty Actuarial Society, Volume LXXVIII, 1991

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RISK LOAD FORMULAS AND PARAMETERS

The following are the formulas underlying ISO's risk load model.

The risk load formulas incorporate parameter risk using a parameter transformation. In the following formulas, we use the notation AVSEV(PL,α) and SECM(PL,α) to represent the limited moments of a transformed loss size distribution. The distribution is transformed by multiplying all occurrences by the constant "α". AVSEV represents the limited average severity and SECM represents the limited second moment of the transformed distribution. The following formulas represent an approximation of the effect of parameter risk on the severity distribution:

$$\text{AVSEV}(\text{PL}, \alpha) = \alpha \times \text{LAS}(\text{PL}/\alpha)$$

$$\text{SECM}(\text{PL}, \alpha) = \alpha^2 \times \text{SECM}(\text{PL}/\alpha)$$

The formulas for the LAS(PL) and SECM(PL) of a mixed exponential are as follows:

$$\text{LAS(PL)} = \sum_i w_i\, \mu_i\, [\, 1 - \exp(-\text{PL} / \mu_i)\,]$$

$$\text{SECM(PL)} = \sum_i 2\, w_i\, \mu_i^2 \left[1 - \left(1 + \frac{\text{PL}}{\mu_i} \right) \exp\left(-\frac{\text{PL}}{\mu_i} \right) \right]$$

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RISK LOAD FORMULAS AND PARAMETERS

(1) *Total Risk Load*
The vector of risk load amounts for a particular increased limits table, R, is:

$$\mathbf{R} = \lambda[\mathbf{U} + 2(\mathbf{V}^{a}\cdot\bar{\mathbf{n}}^{a} + \mathbf{V}^{c}\cdot\bar{\mathbf{n}}^{c})]$$

where

λ = the factor which reflects the overall impact of risk load over General and Commercial Automobile Liability. ISO selected this parameter so that the average increased limit factor with risk load divided by the average increased limit factor without risk load equals 1.06.

$\mathbf{U}$ = the vector of risk elements corresponding to process risk. Its j^{th} component is u_j, corresponding to the j^{th} policy limit.

$\mathbf{V}^{a}$ = the matrix describing severity parameter risk.

$\mathbf{V}^{c}$ = the matrix describing frequency parameter risk.

Premises/Operations Liability (state group):

$\bar{\mathbf{n}}^{a}$ = the vector of the expected number of occurrences per insurer in the particular increased limits table (within its state group). The j^{th} component of $\bar{\mathbf{n}}^{a}$ is computed as follows: the basic limit loss weight for that policy limit in the increased limits table (as a percentage) is multiplied by nbara, the expected number of occurrences per insurer per state group, in the particular increased limits table, for all limits combined.

Premises/Operations Liability (state group):

$\bar{\mathbf{n}}^{c}$ = the vector of the expected average number of occurrences per insurer per state for all tables combined. The j^{th} component of $\bar{\mathbf{n}}^{c}$ is computed as follows: the basic limit loss weight for that policy limit in the increased limits table (as a percentage) is multiplied by the Premises/Operations nbarc, which is the expected average number of occurrences per insurer per state for all tables and limits combined.

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RISK LOAD FORMULAS AND PARAMETERS

(2) *Process Risk Load*

The process risk component of the risk load is given by $\lambda\times\mathbf{U}$. The component u_j, associated with the j^{th} limit, is:

$$u_j = E_\alpha[SECM(PL_j,\alpha)] + d\cdot E_\alpha[AVSEV(PL_j,\alpha)^2]$$

where:

α = random variable with mean 1 and variance a. α represents severity parameter risk.
a = .001 (based on a special ISO study).
1 + d = variance-to-mean ratio for occurrence count distribution, contingent on parameters being known. (In other words, if there were no frequency parameter risk, the variance-to-mean ratio would be 1+d.)
E_α = expected value across all values of the parameter α.

Let: $\alpha_1 = 1 - \sqrt{3a}$; $\alpha_2 = 1$; $\alpha_3 = 1 + \sqrt{3a}$;

The Gauss-Hermite approximation[2] provides a discrete approximation for the expected value of a function $G(\alpha)$ across all values of the normally distributed random variable α:

$$E_\alpha[G(\alpha)] = (1/6)G(\alpha_1) + (2/3)G(\alpha_2) + (1/6)G(\alpha_3)$$

for any function $G(\alpha)$.

(3) *Parameter Risk Load*

The parameter risk component of the risk load is given by $\lambda\times 2\times(\mathbf{V}^c\cdot\bar{\mathbf{n}}^c + \mathbf{V}^a\cdot\bar{\mathbf{n}}^a)$.

Evaluation of $\mathbf{V}^c$

v^c_{ij} = element of $\mathbf{V}^c$ corresponding to i^{th} limit, j^{th} limit
= $c\times E_\alpha[AVSEV(PL_i,\alpha)\cdot AVSEV(PL_j,\alpha)]$
c = parameter quantifying frequency parameter risk ("c" does for frequency what "a" does for severity). Values vary by line based on a special ISO study.

Evaluation of $\mathbf{V}^a$

v^a_{ij} = element of $\mathbf{V}^a$ corresponding to i^{th} limit, j^{th} limit
= $E_\alpha[AVSEV(PL_i,\alpha)\cdot AVSEV(PL_j,\alpha)] - E_\alpha[AVSEV(PL_i,\alpha)]\cdot E_\alpha[AVSEV(PL_j,\alpha)]$

[2] A. Ralston, *A First Course in Numerical Analysis*, McGraw-Hill, 1965

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SUMMARY

In summary, we calculate limited average severities from a continuous model of occurrence size. In this model, we fit mixed exponential distributions to trended lag-weighted occurrence-size distributions.

We calculate allocated loss adjustment expense per occurrence that does not vary by policy limit. We calculate unallocated loss adjustment expense by limit as a percentage of the sum of the limited average severity and allocated loss adjustment expense. We calculate risk load amounts reflecting process and parameter risk.

Finally, we calculate the sum of the limited average severity, allocated loss adjustment expense, unallocated loss adjustment expense and risk load. The ratio of this sum at the limit desired to this sum at the basic limit is the per occurrence increased limit factor.

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MANUAL PAGES

REVISED INCREASED LIMIT FACTORS

(Limits are in thousands)

RULE 56.
INCREASED LIMITS TABLES

1. Premises/Operations (Subline Code 334) Table 1 - $100/200 Basic Limit

	Per Occurrence						
Aggregate	**$ 25**	**50**	**100**	**200**	**300**	**500**	**1,000**
$ 50	0.72	0.81					
100	0.73	0.84~~0.85~~	0.96~~0.97~~				
200	0.74	0.85~~0.86~~	1.00	1.14~~1.13~~			
300	0.75	0.86~~0.87~~	1.01	1.15~~1.14~~	1.24~~1.22~~		
500		0.88~~0.89~~	1.03	1.17~~1.16~~	1.26~~1.24~~	1.38~~1.36~~	
600		0.89~~0.90~~	1.04	1.18~~1.17~~	1.27~~1.25~~	1.39~~1.37~~	
1,000			1.05	1.19~~1.18~~	1.28~~1.26~~	1.40~~1.38~~	1.52~~1.51~~
1,500				1.20~~1.19~~	1.29~~1.27~~	1.41~~1.39~~	1.53~~1.52~~
2,000				1.21~~1.20~~	1.30~~1.28~~	1.42~~1.40~~	1.54~~1.53~~
2,500					1.31~~1.29~~	1.43~~1.41~~	1.55~~1.54~~
3,000					1.32~~1.30~~	1.44~~1.42~~	1.56~~1.55~~

The following factors MUST be referred to company before using.

	Per Occurrence							
Aggregate	**$ 500**	**1,000**	**1,500**	**2,000**	**3,000**	**4,000**	**5,000**	**10,000**
$ 1,500			1.60					
2,000			1.61	1.66~~1.67~~				
2,500			1.62	1.67~~1.68~~				
3,000			1.63	1.68~~1.69~~	1.77~~1.78~~			
4,000	1.45~~1.43~~	1.57~~1.56~~	1.64	1.69~~1.70~~	1.78~~1.79~~	1.85~~1.86~~		
5,000	1.46~~1.44~~	1.58~~1.57~~	1.65	1.70~~1.71~~	1.79~~1.80~~	1.86~~1.87~~	1.93	
10,000		1.59~~1.58~~	1.66	1.71~~1.72~~	1.80~~1.81~~	1.87~~1.88~~	1.94	2.14~~2.13~~
20,000								2.15~~2.14~~

Table 56.B.1 Premises/Operations (Subline Code 334) Table 1 - $100/200 Basic Limit

MONTANA
GENERAL LIABILITY INCREASED LIMIT FACTORS

MANUAL PAGES

REVISED INCREASED LIMIT FACTORS

(Limits are in thousands)

RULE 56.
INCREASED LIMITS TABLES

2. Premises/Operations (Subline Code 334) Table 2 - $100/200 Basic Limit

	Per Occurrence						
Aggregate	**$ 25**	**50**	**100**	**200**	**300**	**500**	**1,000**
$ 50	0.71	0.80					
100	0.73~~0.72~~	0.84	0.96				
200	0.74~~0.73~~	0.85	1.00	1.16			
300	0.75~~0.74~~	0.86	1.01	1.18	1.29		
500		0.88	1.03	1.20	1.32~~1.31~~	1.47	
600		0.89	1.04	1.21	1.33~~1.32~~	1.49~~1.48~~	
1,000			1.05	1.22	1.34~~1.33~~	1.51~~1.50~~	1.74~~1.73~~
1,500				1.23	1.35~~1.34~~	1.52~~1.51~~	1.75~~1.74~~
2,000				1.24	1.36~~1.35~~	1.53~~1.52~~	1.76~~1.75~~
2,500					1.37~~1.36~~	1.54~~1.53~~	1.77~~1.76~~
3,000					1.38~~1.37~~	1.55~~1.54~~	1.78~~1.77~~

The following factors MUST be referred to company before using.

	Per Occurrence							
Aggregate	**$ 500**	**1,000**	**1,500**	**2,000**	**3,000**	**4,000**	**5,000**	**10,000**
$ 1,500			1.89					
2,000			1.90	2.01				
2,500			1.91	2.02				
3,000			1.92	2.03	2.18~~2.21~~			
4,000	1.56~~1.55~~	1.79~~1.78~~	1.93	2.04	2.19~~2.22~~	2.32~~2.37~~		
5,000	1.57~~1.56~~	1.80~~1.79~~	1.94	2.05	2.20~~2.23~~	2.33~~2.38~~	2.44~~2.51~~	
10,000		1.81~~1.80~~	1.95	2.06	2.21~~2.24~~	2.34~~2.39~~	2.45~~2.52~~	2.79~~2.93~~
20,000								2.80~~2.94~~

Table 56.B.2 Premises/Operations (Subline Code 334) Table 2 - $100/200 Basic Limit

MONTANA
GENERAL LIABILITY INCREASED LIMIT FACTORS

MANUAL PAGES

REVISED INCREASED LIMIT FACTORS

(Limits are in thousands)

RULE 56.
INCREASED LIMITS TABLES

3. Premises/Operations (Subline Code 334) Table 3 - $100/200 Basic Limit

	Per Occurrence						
Aggregate	**$ 25**	**50**	**100**	**200**	**300**	**500**	**1,000**
$ 50	0.75	0.82					
100	0.76	0.85	0.96				
200	0.77	0.86	1.00	1.15			
300	0.78	0.87	1.01	1.18~~1.17~~	1.29		
500		0.89	1.03	1.20~~1.19~~	1.33~~1.32~~	1.50~~1.49~~	
600		0.90	1.04	1.21~~1.20~~	1.34~~1.33~~	1.53~~1.51~~	
1,000			1.05	1.22~~1.21~~	1.36~~1.34~~	1.56~~1.53~~	1.87~~1.82~~
1,500				1.23~~1.22~~	1.37~~1.35~~	1.57~~1.54~~	1.91~~1.83~~
2,000				1.24~~1.23~~	1.38~~1.36~~	1.58~~1.55~~	1.92~~1.84~~
2,500					1.39~~1.37~~	1.59~~1.56~~	1.93~~1.85~~
3,000					1.40~~1.38~~	1.60~~1.57~~	1.94~~1.86~~

The following factors MUST be referred to company before using.

	Per Occurrence							
Aggregate	**$ 500**	**1,000**	**1,500**	**2,000**	**3,000**	**4,000**	**5,000**	**10,000**
$ 1,500			2.12~~2.01~~					
2,000			2.15~~2.02~~	2.30~~2.15~~				
2,500			2.16~~2.03~~	2.32~~2.16~~				
3,000			2.17~~2.04~~	2.33~~2.17~~	2.54~~2.37~~			
4,000	1.61~~1.58~~	1.95~~1.87~~	2.18~~2.05~~	2.34~~2.18~~	2.57~~2.38~~	2.72~~2.54~~		
5,000	1.62~~1.59~~	1.96~~1.88~~	2.19~~2.06~~	2.35~~2.19~~	2.58~~2.39~~	2.74~~2.55~~	2.87~~2.68~~	
10,000		1.97~~1.89~~	2.20~~2.07~~	2.36~~2.20~~	2.59~~2.40~~	2.76~~2.56~~	2.90~~2.70~~	3.40~~3.19~~
20,000								3.42~~3.20~~

Table 56.B.3 Premises/Operations (Subline Code 334) Table 3 - $100/200 Basic Limit